The closure Problem

A statistical approach is used because turbulence consists of random fluctuations of the various flow properties. Reynolds (1895) introduced the procedure by in which all quantities are expressed as the sum of mean and fluctuating parts. Hence, the mean of the continuity and Navier Stokes equations is formed term by term.

The nonlinearity of the Navier-Stokes equation leads to the appearance of momentum fluxes (unknowna priori) that actas apparent stresses throughout the flow. Equations for these stresses, which include additionalunknown quantities,are derived. This illustrates the issue of closure, i.e., establishing asufficient number of equations for all of the unknowns.

A discussion of turbulence scales and more-advanced statistical concepts will be considered. To illustrate the nature of turbulence statistics, it is instructive to observehow the velocity field behaves for a turbulent flow.

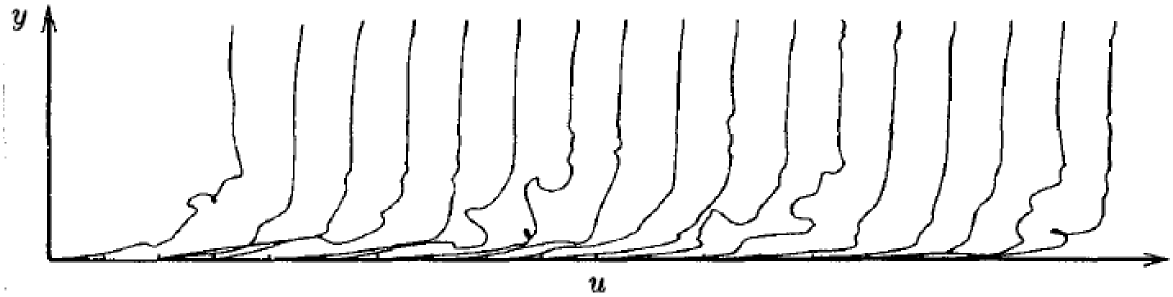
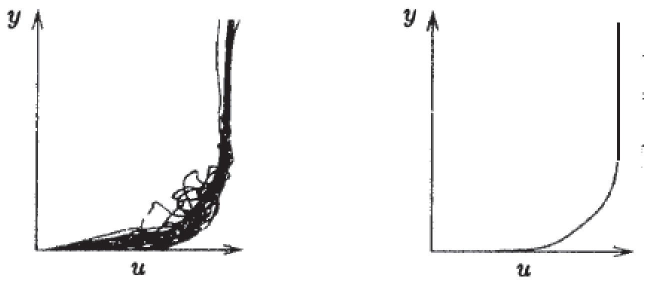


Figure shows Instantaneous boundary-layer velocity profiles at the same distancefrom the leading edge of a flat plate at 17 different instantsusing the hydrogen-bubble technique. The profiles areshown with a series of staggered origins. They appear incorrectly multivalued in a few locations, the measuredvelocity profiles correctly show that the velocity profile changes shape ratherdramatically from one instant to the next.



(a) All profiles – one origin (b) Average profile

The left figure displays all of the velocity profiles, only this time with a commonorigin. Clearly, there is a large scatter in the value of the velocity at eachdistance y from the surface. The right figure shows a standard mean velocity profilefor a boundary layer at the same Reynolds number. Comparison of the profilesin (a) and (b) clearly illustrates that the turbulent fluctuations in the velocitycannot be regarded as a small perturbation relative to the mean value.

Reynolds Averaging

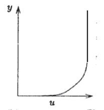
The averaging concepts were introduced by Reynolds (1895). In general,Reynolds averaging assumes a variety of forms involving either an integral or a summation. The general termused to describe these averaging processes is "mean."

The three forms most pertinent in turbulence-model research are:

1. The time average
2. The spatial average
3. The ensemble average

1. Time averaging
It is appropriate for stationary turbulence;a turbulentflow that, on the average, does not vary with time, such as flow in a pipe drivenby a constant-speed blower.

For such a flow, we express an instantaneous flow variable as f(x, t). Its time average, $F_T(x)$, is defined by



(b) Average profile

$$F_T(\mathbf{x}) = \lim_{T\to\infty} \frac{1}{T} \int_t^{t+T} f(\mathbf{x}, t)\, dt$$

This velocity profile was obtained using time averaging for accurate measurements of a similar boundary layer.
The applicability of Reynolds averaging depends upon this steadiness of mean values.
Time averaging is the most commonly used form of Reynolds averaging because most turbulent flows of interest in engineering are stationary.

2. Spatial averaging
It can be used for homogeneous turbulence, which is aturbulent flow that, on the average, is uniform in all directions. It is averaged overall spatial coordinates by doing a volume integral.

$$F_V(t) = \lim_{V\to\infty} \frac{1}{V} \iiint_V f(\mathbf{x}, t)\, dV$$

3. Ensemble averaging
It is the most general type of Reynolds averaging suitableforflows that decay in time.
From N identical experiments (with initial and boundary conditionsthat differ by random infinitesimal perturbations)

where $f(\mathbf{x}, t) = f_n(\mathbf{x}, t)$ in the n^{th} experiment,

the average is F_E, defined by

$$F_E(\mathbf{x}, t) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} f_n(\mathbf{x}, t)$$

From this point on, only time averaging will be considered. Moreover, consider a stationary turbulent flow.

$$F_T(\mathbf{x}) = \lim_{T\to\infty} \frac{1}{T} \int_t^{t+T} f(\mathbf{x}, t)\, dt$$

For such a flow, the instantaneous velocityis expressed.

$$u_i(\mathbf{x}, t) = U_i(\mathbf{x}) + u_i'(\mathbf{x}, t)$$

The quantity $U_i(\mathbf{x})$ is the time-averaged, or mean, velocity defined by

$$U_i(\mathbf{x}) = \lim_{T\to\infty} \frac{1}{T} \int_t^{t+T} u_i(\mathbf{x}, t)\, dt$$

The time average of the mean velocity is again the same time-averaged value,

$$\overline{U_i}(\mathbf{x}) = \lim_{T\to\infty} \frac{1}{T} \int_t^{t+T} U_i(\mathbf{x})\, dt = U_i(\mathbf{x})$$

where an overbar is shorthand for the time average.
The time average of the fluctuating part of the velocity is zero.

$$\overline{u_i'} = \lim_{T\to\infty} \frac{1}{T} \int_t^{t+T} [u_i(\mathbf{x}, t) - U_i(\mathbf{x})]\, dt = U_i(\mathbf{x}) - \overline{U_i}(\mathbf{x}) = 0$$

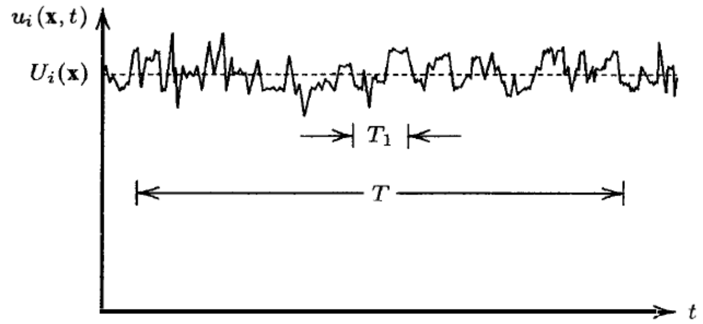


Figure shows the time averaging for stationary turbulence. Although covered by thescale of the graph, the instantaneous velocity, $u_i(x, t)$, has continuous derivativesof all order.

- While the above equations are mathematically well defined, it can never truly be realized infinite T in any physical flow. This is not a serious problem in practice.
- In establishing time average, as illustrated in figure, a time T is just selected that is very long relative to the maximum period of the velocity fluctuations, T_1.

An example,

For flow at 10 m/sec in a 5 cm diameter pipe, anintegration time of 20 seconds would probably be adequate. In this time the flowmoves 4000 pipe diameters.

There are some flows for which the mean flow contains very slow variationswith time that are not turbulent in nature. For instance, we might impose aslowly varying periodic pressure gradient in a duct or we might wish to computeflow over a helicopter blade or flow through an automobile muffler.

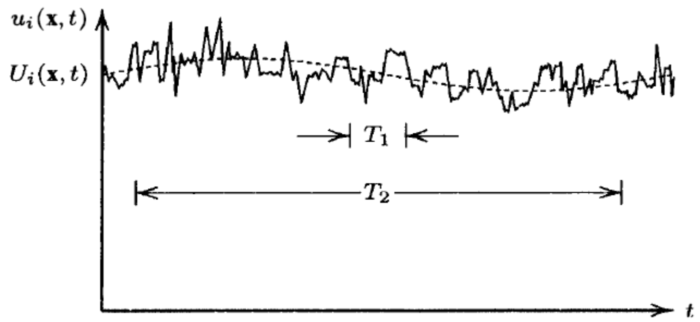


Figure shows the time averaging for non-stationary turbulence. Although covered by thescale of the graph, the instantaneous velocity, $u_i(x, t)$, has continuous derivativesof all order.

$$u_i(\mathbf{x},t) = U_i(\mathbf{x},t) + u_i'(\mathbf{x},t)$$
$$U_i(\mathbf{x},t) = \frac{1}{T}\int_t^{t+T} u_i(\mathbf{x},t)\,dt, \qquad T_1 \ll T \ll T_2$$

where T_2 is the time scale characteristic of the slow variations in the flow. It is implicitlyassuming that time scales T_1 and T_2 exist that differ by several orders of magnitude.Many unsteady flows of engineering interest do not satisfy this condition. Theseequations cannot be usedfor such flows because there is no distinctboundary between our imposed unsteadiness and turbulent fluctuations.

Forsuch flows, the mean and fluctuating componentsare correlated, i.e., the time average of their product is non-vanishing.

Correlations

Thus far averages of linear quantities are considered. When theproduct of two properties is averaged, say Φ and Ψ, we have the following:

$$\overline{\phi\psi} = \overline{(\Phi+\phi')(\Psi+\psi')} = \overline{\Phi\Psi + \Phi\psi' + \Psi\phi' + \phi'\psi'} = \Phi\Psi + \overline{\phi'\psi'}$$

There is a fact that the product of a mean quantity and afluctuating quantity has zero mean because the mean of the latter is zero.

$$\overline{\Phi\psi'} = 0 \;\&\; \overline{\Psi\phi'} = 0$$

Thereis no a priori reason for the mean of the product of two fluctuating quantitiesto vanish.

$$\overline{\phi'\psi'} \neq 0$$

the mean value of a product, $\overline{\phi\psi}$,
differs from the product of the mean values, $\Phi\Psi$.

$$\Phi\Psi \neq \overline{\phi\psi}$$

The quantities ϕ' and ψ' are said to be **correlated** if $\overline{\phi'\psi'} \neq 0$.
They are **uncorrelated** if $\overline{\phi'\psi'} = 0$.

Similarly, for a triple product

$$\overline{\phi\psi\xi} = \Phi\Psi\Xi + \overline{\phi'\psi'}\Xi + \overline{\psi'\xi'}\Phi + \overline{\phi'\xi'}\Psi + \overline{\phi'\psi'\xi'}$$

Again, terms linear in ϕ', ψ' or ξ' have zero mean.
As with terms quadratic in fluctuating quantities,
there is no a priori reason for the cubic term, $\overline{\phi'\psi'\xi'}$, to vanish.